1. Show that the sequence \( a_n = -n + 2 \) is a solution to the recurrence relation \( a_n = a_{n-1} + 2a_{n-2} + 2n - 9 \)

2. Find a recurrence relation satisfied by the sequence: \( a_n = n^2 + 6n \)

3. Using a proof by mathematical induction, show that the following is true for all integers \( n \geq 1 \)

\[
1 \cdot 1! + 2 \cdot 2! + ... + n \cdot n! = (n + 1)! - 1
\]
1 Question 1

Show that the sequence \( a_n = -n + 2 \) is a solution to the recurrence relation \( a_n = a_{n-1} + 2a_{n-2} + 2n - 9 \)

\[
\begin{align*}
a_n &= -n + 2 \\
a_n &= a_{n-1} + 2a_{n-2} + 2n - 9 \\
a_{n-1} &= -(n - 1) + 2 = -n + 1 + 2 = -n + 3 \\
a_{n-2} &= -(n - 2) + 2 = -n + 2 + 2 = -n + 4 \\
a_n &= -n + 3 - 2n + 8 + 2n - 9 = -n + 2 \\
\end{align*}
\]

2 Question 2

Find a recurrence relation satisfied by the sequence: \( a_n = n^2 + 6n \)

\[
\begin{align*}
a_n &= n^2 + 6n \\
a_{n-1} &= (n - 1)^2 + 6n \\
a_{n-1} &= n^2 - 2n + 1 + 6(n - 1) \\
a_{n-1} &= n^2 + 6n - 2n + 1 - 6 \\
a_{n-1} &= a_n - 2n - 5 \\
a_n &= a_{n-1} + 2n + 5 \\
\end{align*}
\]

3 Question 3

Using a proof by mathematical induction, show that the following is true for all integers \( n \geq 1, 1 \cdot 1! + 2 \cdot 2! + \cdots + n \cdot n! = (n + 1)! - 1 \)

**Basis Step**: Show \( P(1) \)

\( P(1) := 1 \cdot 1! = 1 \)

Also, \( (1 + 1)! - 1 = 1 \)

\( \therefore P(1) \)

**Inductive Step**: Show \( P(k) \rightarrow P(k + 1) \)

\( P(k) : 1 \cdot 1! + 2 \cdot 2! + \cdots + k \cdot k! = (k + 1)! - 1 \)

\( P(k + 1) : 1 \cdot 1! + 2 \cdot 2! + \cdots + k \cdot k! + (k + 1)(k + 1)! = (k + 2)! - 1 \)

\( P(k + 1) : (k + 1)! - 1 + (k + 1)(k + 1)! \)

\( P(k + 1) : (k + 1)!(k + 1 + 1) - 1 \)

\( P(k + 1) : (k + 1)!(k + 2) - 1 \)

\( P(k + 1) : (k + 2)! - 1 \)

\( \therefore P(k) \Rightarrow P(k + 1) \)

\( \therefore P(n) \) by mathematical induction