26. a) There are 10 ways to choose the first digit, 9 ways to choose the second, and so on; therefore the answer is \(10 \cdot 9 \cdot 8 \cdot 7 = 5040\).

b) There are 10 ways to choose each of the first three digits and 5 ways to choose the last; therefore the answer is \(10^3 \cdot 5 = 5000\).

c) There are 4 ways to choose the position that is to be different from 9, and 9 ways to choose the digit to go there. Therefore there are \(4 \cdot 9 = 36\) such strings.
34. In each case the answer is \( n^{10} \), where \( n \) is the number of elements in the codomain, since there are \( n \) choices for a function value for each of the 10 elements in the domain.

a) \( 2^{10} = 1024 \)  
   b) \( 3^{10} = 59,049 \)  
   c) \( 4^{10} = 1,048,576 \)  
   d) \( 5^{10} = 9,765,625 \)
50. There are $2^5$ strings that begin with two 0’s (since there are two choices for each of the last five bits). Similarly there are $2^4$ strings that end with three 1’s. Furthermore, there are $2^2$ strings that both begin with two 0’s and end with three 1’s (since only bits 3 and 4 are free to be chosen). By the inclusion–exclusion principle, there are $2^5 + 2^4 - 2^2 = 44$ such strings in all.
20. a) There are $C(10, 3)$ ways to choose the positions for the 0’s, and that is the only choice to be made, so the answer is $C(10, 3) = 120$.

b) There are more 0’s than 1’s if there are fewer than five 1’s. Using the same reasoning as in part (a), together with the sum rule, we obtain the answer $C(10, 0) + C(10, 1) + C(10, 2) + C(10, 3) + C(10, 4) = 1 + 10 + 45 + 120 + 210 = 386$. Alternatively, by symmetry, half of all cases in which there are not five 0’s have more 0’s than 1’s; therefore the answer is $(2^{10} - C(10, 5)/2 = (1024 - 252)/2 = 386$.

c) We want the number of bit strings with 7, 8, 9, or 10 1’s. By the same reasoning as above, there are $C(10, 7) + C(10, 8) + C(10, 9) + C(10, 10) = 120 + 45 + 10 + 1 = 176$ such strings.

d) If a string does not have at least three 1’s, then it has 0, 1, or 2 1’s. There are $C(10, 0) + C(10, 1) + C(10, 2) = 1 + 10 + 45 = 56$ such strings. There are $2^{10} = 1024$ strings in all. Therefore there are $1024 - 56 = 968$ strings with at least three 1’s.
28. a) This is just a matter of choosing 10 players from the group of 13, since we are not told to worry about what positions they play; therefore the answer is \( C(13, 10) = 286 \).

b) This is the same as part (a), except that we need to worry about the order in which the choices are made, since there are 10 distinct positions to be filled. Therefore the answer is \( P(13, 10) = 13!/3! = 1,037,836,800 \).

c) There is only one way to choose the 10 players without choosing a woman, since there are exactly 10 men. Therefore (using part (a)) there are \( 286 - 1 = 285 \) ways to choose the players if at least one of them must be a woman.
32. a) There are $C(16, 5)$ ways to select a committee if there are no restrictions. There are $C(9, 5)$ ways to select a committee from just the 9 men. Therefore there are $C(16, 5) - C(9, 5) = 4368 - 126 = 4242$ committees with at least one woman.

b) There are $C(16, 5)$ ways to select a committee if there are no restrictions. There are $C(9, 5)$ ways to select a committee from just the 9 men. There are $C(7, 5)$ ways to select a committee from just the 7 men. These two possibilities do not overlap, since there are no ways to select a committee containing neither men nor women. Therefore there are $C(16, 5) - C(9, 5) - C(7, 5) = 4368 - 126 - 21 = 4221$ committees with at least one woman and at least one man.
34. a) The only reasonable way to do this is by subtracting from the number of strings with no restrictions the number of strings that do not contain the letter $a$. The answer is $26^6 - 25^6 = 308915776 - 244140625 = 64,775,151$.

b) If our string is to contain both of these letters, then we need to subtract from the total number of strings the number that fail to contain one or the other (or both) of these letters. As in part (a), $25^6$ strings fail to contain an $a$; similarly $25^6$ fail to contain a $b$. This is overcounting, however, since $24^6$ fail to contain both of these letters. Therefore there are $25^6 + 25^6 - 24^6$ strings that fail to contain at least one of these letters. Therefore the answer is $26^6 - (25^6 + 25^6 - 24^6) = 308915776 - (244140625 + 244140625 - 191102976) = 11,737,502$.

c) First choose the position for the $a$; this can be done in 5 ways, since the $b$ must follow it. There are four remaining positions, and these can be filled in $P(24, 4)$ ways, since there are 24 letters left (no repetitions being allowed this time). Therefore the answer is $5P(24, 4) = 1,275,120$.

d) First choose the positions for the $a$ and $b$; this can be done in $C(6, 2)$ ways, since once we pick two positions, we put the $a$ in the left-most and the $b$ in the other. There are four remaining positions, and these can be filled in $P(24, 4)$ ways, since there are 24 letters left (no repetitions being allowed this time). Therefore the answer is $C(6, 2)P(24, 4) = 3,825,360$. 