4. a) We have taken the conjunction of two propositions and asserted one of them. This is, according to Table 1, simplification.

b) We have taken the disjunction of two propositions and the negation of one of them, and asserted the other. This is, according to Table 1, disjunctive syllogism. See Table 1 for the other parts of this exercise as well.

c) modus ponens       d) addition       e) hypothetical syllogism
10. a) If we use modus tollens starting from the back, then we conclude that I am not sore. Another application of modus tollens then tells us that I did not play hockey.

b) We really can’t conclude anything specific here.

c) By universal instantiation, we conclude from the first conditional statement by modus ponens that dragonflies have six legs, and we conclude by modus tollens that spiders are not insects. We could say using existential generalization that, for example, there exists a non-six-legged creature that eats a six-legged creature, and that there exists a non-insect that eats an insect.

d) We can apply universal instantiation to the conditional statement and conclude that if Homer (respectively, Maggie) is a student, then he (she) has an Internet account. Now modus tollens tells us that Homer is not a student. There are no conclusions to be drawn about Maggie.

Section 1.6  Rules of Inference

e) The first conditional statement is that if $x$ is healthy to eat, then $x$ does not taste good. Universal instantiation and modus ponens therefore tell us that tofu does not taste good. The third sentence says that if you eat $x$, then $x$ tastes good. Therefore the fourth hypothesis already follows (by modus tollens) from the first three. No conclusions can be drawn about cheeseburgers from these statements.

f) By disjunctive syllogism, the first two hypotheses allow us to conclude that I am hallucinating. Therefore by modus ponens we know that I see elephants running down the road.
16.  a) This is correct, using universal instantiation and modus tollens.

   b) This is not correct. After applying universal instantiation, it contains the fallacy of denying the hypothesis.

   c) After applying universal instantiation, it contains the fallacy of affirming the conclusion.

   d) This is correct, using universal instantiation and modus ponens.
16. Assume to the contrary that $x$, $y$, and $z$ are all even. Then there exist integers $a$, $b$, and $c$ such that $x = 2a$, $y = 2b$, and $z = 2c$. But then $x + y + z = 2a + 2b + 2c = 2(a + b + c)$ is even by definition. This contradicts the hypothesis that $x + y + z$ is odd. Therefore the assumption was wrong, and at least one of $x$, $y$, and $z$ is odd.

18. We give a proof by contraposition. If it is not true than $m$ is even or $n$ is even, then $m$ and $n$ are both odd. By Exercise 6, this tells us that $mn$ is odd, and our proof is complete.

20. a) We must prove the contrapositive: If $n$ is odd, then $3n + 2$ is odd. Assume that $n$ is odd. Then we can write $n = 2k + 1$ for some integer $k$. Then $3n + 2 = 3(2k + 1) + 2 = 6k + 5 = 2(3k + 2) + 1$. Thus $3n + 2$ is two times some integer plus 1, so it is odd.

b) Suppose that $3n + 2$ is even and that $n$ is odd. Since $3n + 2$ is even, so is $3n$. If we add subtract an odd number from an even number, we get an odd number, so $3n - n = 2n$ is odd. But this is obviously not true. Therefore our supposition was wrong, and the proof by contradiction is complete.
28. We need to prove two things, since this is an “if and only if” statement. First let us prove directly that if \( n \) is even then \( 7n + 4 \) is even. Since \( n \) is even, it can be written as \( 2k \) for some integer \( k \). Then \( 7n + 4 = 14k + 4 = 2(7k + 2) \). This is 2 times an integer, so it is even, as desired. Next we give a proof by contraposition that if \( 7n + 4 \) is even then \( n \) is even. So suppose that \( n \) is not even, i.e., that \( n \) is odd. Then \( n \) can be written as \( 2k + 1 \) for some integer \( k \). Thus \( 7n + 4 = 14k + 11 = 2(7k + 5) + 1 \). This is 1 more than 2 times an integer, so it is odd. That completes the proof by contraposition.
46. **a)** There is a real number whose cube is \(-1\). This is true, since \(x = -1\) is a solution.

**b)** There is an integer such that the number obtained by adding 1 to it is greater than the integer. This is true—in fact, every integer satisfies this statement.

**c)** For every integer, the number obtained by subtracting 1 is again an integer. This is true.

**d)** The square of every integer is an integer. This is true.

48. In each case we want the set of all values of \(x\) in the domain (the set of integers) that satisfy the given equation or inequality.

**a)** It is exactly the positive integers that satisfy this inequality. Therefore the truth set is \(\{x \in \mathbb{Z} \mid x^3 \geq 1\} = \{x \in \mathbb{Z} \mid x \geq 1\} = \{1, 2, 3, \ldots\} \).
48. In each case we want the set of all values of \( x \) in the domain (the set of integers) that satisfy the given equation or inequality.

a) It is exactly the positive integers that satisfy this inequality. Therefore the truth set is \( \{ x \in \mathbb{Z} \mid x^3 \geq 1 \} = \{ x \in \mathbb{Z} \mid x \geq 1 \} = \{ 1, 2, 3, \ldots \} \).

b) The square roots of 2 are not integers, so the truth set is the empty set, \( \emptyset \).

c) Negative integers certainly satisfy this inequality, as do all positive integers greater than 1. However, \( 0 \not< 0^2 \) and \( 1 \not< 1^2 \). Thus the truth set is \( \{ x \in \mathbb{Z} \mid x < x^2 \} = \{ x \in \mathbb{Z} \mid x \neq 0 \land x \neq 1 \} = \{ \ldots, -3, -2, -1, 2, 3, \ldots \} \).
2. a) \( A \cap B \)  
   b) \( A \cap \overline{B} \), which is the same as \( A \setminus B \)  
   c) \( A \cup B \)  
   d) \( \overline{A} \cup \overline{B} \)